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Recovering the Temporal Release Rate of Pollutant Sources in the River in Two-dimensional and real condition

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ABSTRACT: Over the past three decades, many approaches and methods have been investigated based on the inverse problem solving to recover the temporal release rate of pollutant sources, especially in groundwater. But, number of studies is limited about the rivers; therefore, developing a method which can determine temporal release rate of pollutant sources in the river precisely and at the same time be able to consider the conditions of the flow and bed is promising. In the present study, the inverse solution of the advection-dispersion equation for recovering the temporal release rate of pollutant sources leads to the solution of a linear overdetermined system of equations type of ill-posed problem. Therefore, in this research a numerical model based on the inverse matrix approach based on the Tikhonov regularization method and the results of the superposition principle has been applied to the recovery of the temporal release rate of pollutant sources and the exact time of release of the pollutant from the source. The model has been designed to retrieve the complexity time of multiple pollutant sources in a complex state. Also, the model has been verified using real two-dimensional data of Ohio River located in the United States. Finally, a general and practical framework has been introduced to apply in real condition. Eventually, the computational results were showed that, the inverse model can recover the temporal release rate of pollutant sources using the lowest field and downstream data containing high error rate at each point of the river.

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1. INTRODUCTION

With the boom of manufacturing industries on river banks all over the world, it is vital to devise methods for identifying the behavior of the pollutants released into the rivers.

The use of inverse methods for identifying the time and the spots of abandoned pollutants in rivers dates back to three decades. Previous studies on solving the inverse problem of the pollutant transport equation fall into three categories in terms the method applied, including [1]:

1. Simulation-optimization methods
2. Probabilistic and geostatistical methods
3. Mathematical methods

In the first method, the researchers combine an optimization algorithm with other numerical methods of transport and hydrodynamic equations. In the probabilistic and geostatistical methods, probabilistic and geostatistical distributions is used. In the mathematical methods, the inverse problem is solved only in a mathematical framework.

The majority of the researches done on the inverse solution of the pollutant transport equation over the past three decades focus on groundwater, while the problem of rivers have been largely disregarded. Moreover, despite their strengths, all the researches on the river have some shortcomings. Among these shortcomings are, inapplicability of the method to real conditions, disregard of computational complexities and the absence of a framework for use in real condition. “Real conditions” here means the flow, the river bed, and the patterns of pollutant release in the problem solving environment.

The main objective of this research is to overcome some of these shortcomings and to provide a feasible framework for the applied method. This research differs from the previous ones in procedure and the techniques of problem solving and the method of inducing and presenting the framework. Among the other innovations of this study is its use of more complicated examples (hypothetical and real examples) for solving inverse problem of the transport equation.

2. MATERIALS AND METHODS

2.1. Forward Solution of Hydrodynamic and Transport Equations in Two-Dimensional Domain

In the two-dimensional domain, four categories of hydrodynamic and transport equation must be solved in a forward mode, so that, the inputs of the inverse model can be found to find the final result of the problem, which is the release rate of the pollutant sources. The general form of these equations for the two-dimensional domains under unsteady and non-uniform flow condition is expresses as follows [2, 3]:

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Table 1. Inverse solution results for 3rd observation point

	First-order regularization		Zeroth-order regularization	
Data frequency (min)	10			
Data Error (%)	15	5	15	5
Relative error (%)	2.115	1.005	2.663	1.123
Coefficients Matrix Dimensions	(16×3)			
regularization parameter	0.0011	0.0002	0.0017	0.00025

Table 2. Inverse solution results for 4th observation point

	First-order regularization		Zeroth-order regularization	
Data frequency (min)	25			
Data Error (%)	15	5	15	5
Relative error (%)	2.845	1.393	3.081	1.923
Coefficients Matrix Dimensions	(16×3)			
regularization parameter	0.0018	0.00027	0.0021	0.00034

$$\frac{\partial h}{\partial t}+\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial y}=0 \tag{1}$$

$$\frac{\partial(hu)}{\partial t}+\frac{\partial(huv)}{\partial y}+\frac{\partial}{\partial x}\left(hu^2+\frac{1}{2}gh^2\right)-gh\left(S_{0x}-S_{fx}\right)=0 \tag{2}$$

$$\frac{\partial(hv)}{\partial t}+\frac{\partial(huv)}{\partial x}+\frac{\partial}{\partial y}\left(hv^2+\frac{1}{2}gh^2\right)-gh\left(S_{0y}-S_{fy}\right)=0 \tag{3}$$

$$\frac{\partial(hc)}{\partial t}+\frac{\partial(huc)}{\partial x}+\frac{\partial(hvc)}{\partial y}-\frac{\partial}{\partial x}\left(hD_x\frac{\partial c}{\partial x}\right)-\frac{\partial}{\partial y}\left(hD_y\frac{\partial c}{\partial y}\right)$$
$$+hkc-\sum_{i=1}^{n_S}w_i(t)\delta(x-x_i)\delta(y-y_i)=0 \tag{4}$$

In the above equations, k is the decay rate, h is the flow depth; u is the flow velocity in the direction of x; v is the flow velocity in the direction of y; S_{0x} is the bottom slope in the direction of x; S_{0y} is the bottom slope in the direction of y; S_{fx} is the friction slope in the direction of x; S_{fy} is the friction slope in the direction of y; D_x is the dispersion coefficient in the direction of x; D_y is the dispersion coefficient in the direction of y; $\delta(\ )$ is Dirac delta function; and (x_i, y_i) is the coordinates of the ith pollutant source on the $x-y$ horizontal coordinate plane.

2.2. Inverse Solution of the Pollutant Transport Equation in Two-Dimensional Domain

In this section, we must consider the integral form of the inverse solution of the advection-dispersion equation in a two-dimensional domain. This equation, in case of the existence of n_s pollutant sources, is the Volterra linear integral equation of the first type, which is expressed as follows [1]:

$$c(x,y,t)=\sum_{i=1}^{n_s}\int_0^t w_i(\tau)G(x,y,t;x_i,y_i,\tau)\,d\tau \tag{5}$$

In the above equation, $w_i(\tau)$ is the intensity function of the pollutant source; (x_i, y_i) is the source location; and $c(x,y,t)$ are the values of concentration over time (the values of response curve) in different spots of the river. The integral solution of the above equations means obtaining $w_i(\tau)$ with the assumption that $c(x,y,t)$ and $G(x,y,t;x_i,y_i,\tau)$ are known. The above equation, after discretization and turning it into a linear least-squares problem is solvable. For inverse recovering the temporal release rate of the pollutant source in two-dimensional domain, we should solve the linear system of equation, in which the known variables of the problem is the same as the values of the response curve concentrations in the observation points, the unknowns of problem are average values of the pollutant source loading and the coefficients of the equations are the values of the unit loading concentration at the observation points; In this system, each of these components is defined as a matrix, and the numerical codes to solve the system are written (according to the solution methods for linear system of equation).

3. RESULTS AND DISCUSSION

3.1. Example 1: Recovering the temporal release rate of a Pollutant Source in the Two-Dimensional Domain

In this example, the information of 4 observation points are used to recover the temporal release rate of a step loading

Table 3. inverse solution results for multiple pollutant sources

	First-order regularization		Zeroth-order regularization	
Data Error (%)	15	5	15	5
Relative error (%)	12.53	5.05	13.98	6.79
Coefficients Matrix Dimensions (in complex mode)	(272×50)			
regularization parameter	0.0123	0.00291	0.00953	0.00136

source (at the opening of the river length) and to present the required results. The observation points are located at different distances on the 1600-meter-long river length. In the result announcement section, the results of the inverse solution in 2 observation points are presented in a table. Table 2 shows the results of the inverse solution for the 4th observation point. The results of the inverse solution for the 3rd observation point are presented in Table 1. Since computational error are few, the recovery of the temporal release rate of the pollutant source is done with high accuracy.

3.2. Example 2: Recovering the temporal release rate of Multiple Pollutant Sources in Two-Dimensional Domain (Real Condition)

In this example, to recover the temporal release rate of pollutant sources in the complex mode, the real flow condition and the river bed is considered. The studied area is Ohio River in USA. The real condition of the river has been used to verify the presented inverse model to provide a general framework. The number of pollutant sources in this range from this length of the river are 3 point sources and the number of observation points is one point per each pollutant source.

For the applied model for the pollutant sources, we have tried to use all the possible states of loading models. As for the pollutant release time, the location of the pollutant sources and their distances from each other, we have tried to make the conditions as real as possible. Based on Table 3, recovering the temporal release rate of multiple pollutant sources in this domain (real condition) has been done in one stage, using a minimum Amount of inverse model run. This model is applicable to countless pollutant sources and observation points.

4. CONCLUSION

So far, various methods have been used to recover the location and the time of release from pollutant sources, and in any one of them some conditions have been considered as the hypotheses. In past researches, the inverse problem in river has seldom been solved via real data and under real and complicated environment. Therefore, it is necessary to call for researches that can overcome the limitations of past researches. The following is an outline of the general framework for the forward and inverse solution of the transport equation to recover the temporal release rate of pollutant sources in two-dimensional domain in river.

1. Based on Example 1 (Hypothetical), by testing the different values of dispersion coefficient and different loadings (with the minimum loading time being 30 minutes), the best results of the inverse solving will be obtained with the relative error rate of less than 5%.

2. Based on Example 2 (real), for the river widths up to 480 meters and the loadings of 4 to 7 hours, if the Peclet number is located in the number range (3.05-6.3), the best results of the inverse solving will be obtained with the error rate of less than 15%.

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